

Package ‘primes’

January 9, 2024

Type Package

Title Fast Functions for Prime Numbers

Version 1.6.0

Date 2023-12-30

Description Fast functions for dealing with prime numbers, such as testing whether a number is prime and generating a sequence prime numbers. Additional functions include finding prime factors and Ruth-Aaron pairs, finding next and previous prime numbers in the series, finding or estimating the nth prime, estimating the number of primes less than or equal to an arbitrary number, computing primorials, prime k-tuples (e.g., twin primes), finding the greatest common divisor and smallest (least) common multiple, testing whether two numbers are coprime, and computing Euler's totient function. Most functions are vectorized for speed and convenience.

License MIT + file LICENSE

Depends R (>= 4.0)

Imports Rcpp

LinkingTo Rcpp

Suggests testthat

URL <https://github.com/ironholds/primes>

BugReports <https://github.com/ironholds/primes/issues>

RoxygenNote 7.2.3

Encoding UTF-8

LazyData true

NeedsCompilation yes

Author Os Keyes [aut, cre],
Paul Egeler [aut] (<<https://orcid.org/0000-0001-6948-9498>>)

Maintainer Os Keyes <ironholds@gmail.com>

Repository CRAN

Date/Publication 2024-01-09 00:40:05 UTC

R topics documented:

gcd	2
generate_n_primes	3
is_prime	4
k_tuple	5
next_prime	6
nth_prime	7
phi	8
primes	9
prime_count	9
prime_factors	10
primorial	11
ruth_aaron_pairs	11

Index	13
--------------	-----------

gcd	<i>Find the Greatest Common Divisor, Smallest Common Multiple, or Coprimality</i>
-----	---

Description

These functions provide vectorized computations for the greatest common divisor (gcd), smallest common multiple (scm), and coprimality. Coprime numbers are also called *mutually prime* or *relatively prime* numbers. The smallest common multiple is often called the *least common multiple*.

Usage

gcd(m, n)

scm(m, n)

coprime(m, n)

Rgcd(...)

Rscm(...)

Arguments

m, n, ... integer vectors.

Details

The greatest common divisor uses Euclid's algorithm, a fast and widely used method. The smallest common multiple and coprimality are computed using the gcd, where $scm = \frac{a}{gcd(a,b)} \times b$ and two numbers are coprime when $gcd = 1$.

The gcd, scm, and coprime functions perform element-wise computation. The Rgcd and Rscm functions perform gcd and scm over multiple values using reduction. That is, they compute the greatest common divisor and least common multiple for an arbitrary number of integers based on the properties $gcd(a_1, a_2, \dots, a_n) = gcd(gcd(a_1, a_2, \dots), a_n)$ and $scm(a_1, a_2, \dots, a_n) = scm(scm(a_1, a_2, \dots), a_n)$. The binary operation is applied to two elements; then the result is used as the first operand in a call with the next element. This is done iteratively until all elements are used. It is idiomatically equivalent to Reduce(gcd, x) or Reduce(scm, x), where x is a vector of integers, but much faster.

Value

The functions gcd, scm, and coprime return a vector of the length of longest input vector. If one vector is shorter, it will be recycled. The gcd and scm functions return an integer vector while coprime returns a logical vector. The reduction functions Rgcd and Rscm return a single integer.

Author(s)

Paul Egeler, MS

Examples

```
gcd(c(18, 22, 49, 13), 42)
## [1] 6 2 7 1
```

```
Rgcd(18, 24, 36, 12)
## [1] 6
```

```
scm(60, 90)
## [1] 180
```

```
Rscm(1:10)
## [1] 2520
```

```
coprime(60, c(77, 90))
## [1] TRUE FALSE
```

generate_n_primes

Generate a Sequence of Prime Numbers

Description

Generate a sequence of prime numbers from min to max or generate a vector of the first n primes. Both functions use a fast implementation of the Sieve of Eratosthenes.

Usage

```
generate_n_primes(n)

generate_primes(min = 2L, max)
```

Arguments

n	the number of primes to generate.
min	the lower bound of the sequence.
max	the upper bound of the sequence.

Value

An integer vector of prime numbers.

Author(s)

Paul Egeler, MS

Examples

```
generate_primes(max = 12)
## [1]  2  3  5  7 11

generate_n_primes(5)
## [1]  2  3  5  7 11
```

is_prime

Test for Prime Numbers

Description

Test whether a vector of numbers is prime or composite.

Usage

```
is_prime(x)
```

Arguments

x	an integer vector containing elements to be tested for primality.
---	---

Value

A logical vector.

Author(s)

Os Keyes and Paul Egeler, MS

Examples

```
is_prime(4:7)
## [1] FALSE TRUE FALSE TRUE
```

```
is_prime(1299827)
## [1] TRUE
```

<i>k_tuple</i>	<i>Prime k-tuples</i>
----------------	-----------------------

Description

Use prime *k*-tuples to create lists of twin primes, cousin primes, prime triplets, and so forth.

Usage

```
k_tuple(min, max, tuple)
sexy_prime_triplets(min, max)
twin_primes(min, max)
cousin_primes(min, max)
sexy_primes(min, max)
third_cousin_primes(min, max)
```

Arguments

<i>min</i>	the lower bound of the sequence.
<i>max</i>	the upper bound of the sequence.
<i>tuple</i>	an integer vector representing the target <i>k</i> -tuple pattern.

Details

You can construct your own tuples and generate series of primes using *k_tuple*; however, there are functions that exist for some of the named relationships. They are listed below.

- *twin_primes*: represents $c(\emptyset, 2)$.
- *cousin_primes*: represents $c(\emptyset, 4)$.

- `third_cousin_primes`: represents $c(0, 8)$.
- `sexy_primes`: represents $c(0, 6)$.
- `sexy_prime_triplets`: represents $c(0, 6, 12)$. (This relationship is unique in that $p + 18$ is guaranteed to be composite.)

The term "third cousin primes" is of the author's coinage. There is no canonical name for that relationship to the author's knowledge.

Value

A list of vectors of prime numbers satisfying the condition of tuple.

Author(s)

Paul Egeler, MS

Examples

```
# All twin primes up to 13
twin_primes(2, 13) # Identical to `k_tuple(2, 13, c(0,2))`
## [[1]]
## [1] 3 5
##
## [[2]]
## [1] 5 7
##
## [[3]]
## [1] 11 13

# Some prime triplets
k_tuple(2, 19, c(0,4,6))
## [[1]]
## [1] 7 11 13
##
## [[2]]
## [1] 13 17 19
```

next_prime

Find the Next and Previous Prime Numbers

Description

Find the next prime numbers or previous prime numbers over a vector.

Usage

```
next_prime(x)
```

```
prev_prime(x)
```

Arguments

x a vector of integers from which to start the search.

Details

For `prev_prime`, if a value is less than or equal to 2, the function will return NA.

Value

An integer vector of prime numbers.

Author(s)

Paul Egeler, MS

Examples

```
next_prime(5)
## [1] 7

prev_prime(5:7)
## [1] 3 5 5
```

`nth_prime` *Get the n-th Prime from the Sequence of Primes.*

Description

Get the n-th prime, p_n , in the sequence of primes.

Usage

```
nth_prime(x)
```

Arguments

x an integer vector.

Value

An integer vector.

Author(s)

Paul Egeler, MS

Examples

```
nth_prime(5)
## [1] 11

nth_prime(c(1:3, 7))
## [1] 2 3 5 17
```

phi

Euler's Totient Function

Description

Compute Euler's Totient Function ($\phi(n)$). Provides the count of k integers that are coprime with n such that $1 \leq k \leq n$ and $\gcd(n, k) = 1$.

Usage

```
phi(n)
```

Arguments

n an integer vector.

Value

An integer vector.

Author(s)

Paul Egeler, MS

References

"Euler's totient function" (2020) Wikipedia. https://en.wikipedia.org/wiki/Euler%27s_totient_function (Accessed 21 Aug 2020).

See Also

[gcd](#), [coprime](#), [prime_factors](#)

Examples

```
phi(12)
## [1] 4

phi(c(9, 10, 142))
## [1] 6 4 70
```

primes	<i>Pre-computed Prime Numbers</i>
--------	-----------------------------------

Description

The first one thousand prime numbers.

Usage

primes

Format

An integer vector containing the first one thousand prime numbers.

See Also

[generate_primes](#), [generate_n_primes](#)

prime_count	<i>Prime-counting Functions and Estimating the Value of the n-th Prime</i>
-------------	--

Description

Functions for estimating $\pi(n)$ —the number of primes less than or equal to n —and for estimating the value of p_n , the n -th prime number.

Usage

prime_count(n, upper_bound)

nth_prime_estimate(n, upper_bound)

Arguments

`n` an integer. See *Details* for more information.

`upper_bound` a logical indicating whether to estimate the lower- or upper bound.

Details

The `prime_count` function estimates the number of primes $\leq n$. When `upper_bound = FALSE`, it is guaranteed to under-estimate for all $n \geq 17$. When `upper_bound = TRUE`, it holds for all positive n .

The `nth_prime_estimate` function brackets upper and lower bound values of the n th prime. It is valid for $n \geq 6$.

The methods of estimation used here are a few of many alternatives. For further information, the reader is directed to the *References* section.

Author(s)

Paul Egeler, MS

References

"Prime-counting function" (2020) *Wikipedia*. https://en.wikipedia.org/wiki/Prime-counting_function#Inequalities (Accessed 26 Jul 2020).

prime_factors

Perform Prime Factorization on a Vector

Description

Compute the prime factors of elements of an integer vector.

Usage

```
prime_factors(x)
```

Arguments

x an integer vector.

Value

A list of integer vectors reflecting the prime factorizations of each element of the input vector.

Author(s)

Paul Egeler, MS

Examples

```
prime_factors(c(1, 5:7, 99))
## [[1]]
## integer(0)
##
## [[2]]
## [1] 5
##
## [[3]]
## [1] 2 3
##
## [[4]]
## [1] 7
##
## [[5]]
## [1] 3 3 11
```

primorial	<i>Compute the Primorial</i>
-----------	------------------------------

Description

Computes the primorial for prime numbers and natural numbers.

Usage

```
primorial_n(n)
```

```
primorial_p(n)
```

Arguments

`n` an integer indicating the numbers to be used in the computation. See *Details* for more information.

Details

The `primorial_p` function computes the primorial with respect to the first n *prime* numbers; while the `primorial_n` function computes the primorial with respect to the first n *natural* numbers.

Value

A numeric vector of length 1.

Author(s)

Paul Egeler, MS

ruth_aaron_pairs	<i>Find Ruth-Aaron Pairs of Integers</i>
------------------	--

Description

Find pairs of consecutive integers where the prime factors sum to the same value. For example, (5, 6) are Ruth-Aaron pairs because the prime factors $5 = 2 + 3$.

Usage

```
ruth_aaron_pairs(min, max, distinct = FALSE)
```

Arguments

<code>min</code>	an integer representing the minimum number to check.
<code>max</code>	an integer representing the maximum number to check.
<code>distinct</code>	a logical indicating whether to consider repeating prime factors or only distinct prime number factors.

Value

A List of integer pairs.

Author(s)

Paul Egeler, MS

Index

* datasets

- primes, 9

- coprime, 8
- coprime (gcd), 2
- cousin_primes (k_tuple), 5

- gcd, 2, 8
- generate_n_primes, 3, 9
- generate_primes, 9
- generate_primes (generate_n_primes), 3

- is_prime, 4

- k_tuple, 5

- lcm (gcd), 2

- next_prime, 6
- nth_prime, 7
- nth_prime_estimate (prime_count), 9

- phi, 8
- prev_prime (next_prime), 6
- prime_count, 9
- prime_factors, 8, 10
- primes, 9
- primorial, 11
- primorial_n (primorial), 11
- primorial_p (primorial), 11

- Rgcd (gcd), 2
- Rscm (gcd), 2
- ruth_aaron_pairs, 11

- scm (gcd), 2
- sexy_prime_triplets (k_tuple), 5
- sexy_primes (k_tuple), 5

- third_cousin_primes (k_tuple), 5
- twin_primes (k_tuple), 5